NHH

INSTITUTT FOR FORETAKSØKONOMI

DEPARTMENT OF FINANCE AND MANAGEMENT SCIENCE

FOR 5 2011

ISSN: 1500-4066

MARCH 2011

Discussion paper

On the Pricing of Performance Sensitive Debt

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April 29, 2012

Abstract

Performance sensitive debt (PSD) contracts link a loan's interest rate to a measure of the borrower's credit relevant performance, e.g., if the borrower becomes less credit worthy, the interest rate increases according to a predetermined schedule. We derive and empirically test a pricing model for PSD contracts and find that *interest increasing* contracts are priced reflecting a substantial risk of shocks to borrower credit quality. Borrowers using such contracts are of an overall higher credit quality compared to borrowers using *interest decreasing* contracts. These contracts are priced as if no risk of shocks to borrower credit quality is present.

JEL Codes: G12, G13, G32

*Earlier versions of this paper have been presented at FIBE 2010, NFN doctoral student workshop (2010), at the University of Stavanger, the Midwest Finance Association's Annual meeting 2011, and internal seminars at NHH. We would like to thank Martin Andersen, Jan Erik Berre, Petter Bjerksund, Neville Crow, B. Espen Eckbo, Ralf Elsas, Maria Paz Espinosa, Hans K. Hvide, Thore Johnsen, Timo Korkeamäki, Terje Lensberg, Bernt Arne Ødegård, Marianne Økland, Anders Øksendal, Petter Osmundsen, Kristian Råum, Karin Thorburn, Erlend Torjussen, and Pietro Veronesi for valuable comments and suggestions. We, especially, thank Gustavo Manso for insightful comments and suggestions. The authors thank Sara Haugen for excellent research assistance and the Finance Market Fund for financial support.

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Introduction

Performance sensitive debt (PSD) contracts link the interest rate paid on a firm's loan to a measure of its credit relevant performance over time. The two most common categories of firm credit performance measures are cash flow ratios and credit ratings. Since the mid 1990's performance sensitive provisions are common in both private and public corporate loans. Using Thomson Reuter's Dealscan database for the years 1993 - 2010 we find that PSD loans constitute 11.2% of the total number of loans in the database and 35.1% of loans granted in the U.S. and Canada. Market participants indicate that more than half of recently issued syndicated bank loans in Europe include such provisions. Based on financial valuation theory, using results from Mjøs & Persson (2010) and observed contractual designs, we propose a valuation model to price PSD contracts with a credit performance measure defined by debt to cash flow (Debt/CF). Our model includes the possibility of shocks to the borrower's credit quality. The source of such a shock could in principle be any economic event that affects the borrower's credit quality. In our valuation model we implement these shocks as jumps in the total amount of debt. We, in particular, study two important subclasses of PSD contracts; *interest increasing* and *interest decreasing* PSD contracts. The first category includes loan contracts where the borrower initially pays the lowest contractual interest rate, and where the interest rate increases if the borrower's credit performance measure deteriorates. The second category includes loan contracts where the borrower pays the highest contractual interest rate initially, and where the interest rate decreases if the borrower's credit performance measure improves.

We empirically test the pricing performance of our model in the following way: Firstly, we collect contractual, borrower and financial market data related to 3,052 PSD contracts. Secondly, we estimate volatility and drift from borrower and market data. Thirdly, we impose *ad hoc* assumptions regarding jumps in the amount of debt. Finally, we calibrate the model prices to par[1] by adjusting the shock frequency. Our results show a significant difference between interest increasing and interest decreasing contracts regarding shocks to credit quality. Shocks to credit quality must be included to correctly price interest increasing PSD contracts. Interest decreasing PSD contracts are priced as if no shocks are present. By analyzing borrower data we find that the most financially sound borrowers choose interest increasing PSD contracts, which are priced reflecting substantially higher risk of shocks to borrower credit quality compared to interest decreasing PSD contracts.

Our paper is related to the article by Manso, Strulovici, & Tchistyi (2010), who develop a general pricing model for a broad class of PSD using the framework of Leland (1994). They show that with no other market imperfections than bankruptcy costs and tax benefits of debt, the use of PSD contracts leads to earlier default and lower equity value compared to comparable fixed-rate debt, and therefore find the use of these contracts not optimal. In their screening model a company can choose to issue either performance sensitive debt or fixed rate debt. They find that the existence of PSD contracts can be explained by the

[1] Our analysis does not take into account potential administrative, capital, and regulatory related costs incurred by the lender and, thus, assumes that the initial market value equals par value of the loan.

contracts' ability to mitigate adverse selection problems for the borrower. This conclusion is supported by an empirical analysis which shows that firms using performance sensitive debt are more likely to get improved credit ratings in the future compared to firms that choose ordinary fixed-interest loans. They do not empirically test their pricing models for PSD contracts, nor do they study interest increasing or interest decreasing contracts separately. Asquith, Beatty, & Weber (2005) study these two PSD categories separately, and find that interest decreasing PSD contracts are used when adverse selection costs are high.

We focus on the pricing of PSD contracts, and our model of a PSD contract differs from Manso et al. (2010) in the following ways. We analyze contracts with finite maturity, and derive a closed form solution in the special case of no shocks in the credit performance measure. All PSD contracts in our sample have a contractual default barrier, defined by a critical value of the credit performance measure. This default barrier is determined as an element of the initial contract negotiations jointly with all other contractual parameters. The contractual default barrier is exogenous (see Table VIII for an example) as opposed to the optimal endogenous company liquidation-trigger analyzed in Manso et al. (2010). A calculation of an optimal endogenous default level requires assumptions about a firm's complete capital structure. In our sample, most borrowers have a complex debt structure, and PSD, on average, only makes up less than half of a company's total debt as illustrated by Table V. It is, thus, an advantage of our analysis that we use the exogenous contractual default barrier directly. In order to better understand the contractual default barrier, we randomly select 50 loan contracts in our sample and manually review the terms of each contract. We find that all of these contracts have cross-default clauses, and have also consulted lawyers in this respect. Cross-default implies that a borrower's default on any single debt contract leads to default on its other debt contracts as well. Whilst acknowledging the impact of PSD contracts on capital structure and debt renegotiations both as related and relevant research topics, they fall outside the scope of this paper.

We only analyze PSD contracts where the credit performance measure is based on total firm cash flow and debt. This assumption excludes, e.g., rating based contracts. Market evidence indicates that cash flow and debt based performance measures, which our model covers, are the most common performance measures in such PSD contracts (See Table I). Our empirical results confirm and refine the understanding of the signaling hypothesis in Manso et al. (2010). Our results indicate that borrowers may signal quality by selecting interest increasing PSD loans. Low quality borrowers, which are more credit constrained, do not mimic this behavior since they would have to incur the additional borrowing costs related to the lenders rational assessment of risk of shocks to credit quality. Contrary to the findings of Asquith et al. (2005), we find that interest decreasing PSD contracts cannot be used for signaling purposes since these contracts do not induce a cost for low quality borrowers.

Several papers have empirically tested the ability of structural debt models to produce correct prices and/or credit spreads, e.g., Eom, Helwege, & Huang (2004), Huang & Huang (2003). Our empirical analysis confirms the initial overpricing in situations with a large distance-to-default as found in earlier literature.

The remainder of the paper is structured as follows. Section 1 discusses related literature. Section 2 presents the general economic set-up and some details of the theoretical pricing model. Section 3 provides a brief description of the market for PSD loans, as well as a description of the data we use. Section 4 includes an example of a PSD contract and calculates its theoretical price in the case of no jumps. In Section 5 we present and analyze empirical results for the whole sample. Section 6 concludes. Technical calculations and supplementary descriptive statistics are collected in two appendices.

1 Related Literature

Our paper is related to the broad literature on credit risk, and especially to the part of the literature focusing on pricing performance sensitive debt. Credit risk is defined as the risk that a borrower will not honor her contractual obligations towards the lender. There are two dominating approaches to model credit risk in the finance literature; *structural models* and *reduced form models*. Structural models view debt and equity as contingent claims on total firm value, and hence these claims could be valued using option pricing techniques. This approach was pioneered by Merton (1974) and further developed by, e.g., Black & Cox (1976), Geske (1977), Longstaff & Schwartz (1995) and Leland (1994). Reduced form models assume that credit risk is modeled by default arrival intensity. This approach was pioneered by Jarrow & Turnbull (1992) and further developed by, e.g., Jarrow & Turnbull (1995) and Jarrow, Lando, & Turnbull (1997). In the recent years researchers have successfully merged the two modeling approaches by introducing alternative information filtrations and jumps in structural debt models. In short, reduced form models can be viewed as structural models with incomplete information or with jumps in the underlying asset. Duffie & Lando (2001) were the first to introduce incomplete information. Further advances have been made by, e.g., Collin-Dufresne, Goldstein, & Helwege (2003), Jarrow & Protter (2004), Coculesco, Geman, & Jeanblanc (2008), Guo, Jarrow, & Zeng (2009), and Lindset, Lund, & Persson (2011). The notion of jump risk in financial economics was introduced by Merton (1976).

Both structural credit risk models and reduced form models have been successfully applied for the purpose of pricing performance sensitive debt contracts. Lando & Mortensen (2004) and Houweling, Mentink, & Vorst (2004) use the latter framework to develop pricing models for rating based PSD. Similarly, Bhanot & Mello (2006) and Koziol & Lawrenz (2010) develop structural pricing models for rating-based PSD contracts. Manso, Strulovici, & Tchistyi (2010) also use the structural model framework to derive a theoretical model pricing more general PSD contracts.

The existing literature on performance sensitive debt has mainly been focused on explaining the existence of these contracts. Assuming positive bankruptcy costs, performance sensitive debt contracts, at least at a first glance, seem inefficient. Whilst increased interest payments in bad states of the world may have an *ex ante* disciplining effect, the *ex post* conditional probability of bankruptcy increases, and a PSD contract destroys, rather than adds, firm value. In the light of this intuition, existing research on PSD has mainly focused on the efficiency and existence of these contracts. Regarding the existence, the problem of potential information asymmetry as pointed out in the seminal work of Myers & Majluf (1984), and the problem of agency costs, identified by Jensen & Meckling (1976), both may be important explanations for the use of PSD contracts as disciplining and/or signaling mechanisms.

Tchistyi (2006) studies optimal security design in a dynamic setting where the agency problem arises from the assumption that a manager in charge of a project could divert cash flows for his own consumption. Allowing cash flows to be correlated over time, he finds that the optimal debt contract could be implemented using a credit line with performance sensitive provisions.

Asquith, Beatty, & Weber (2005) make an important contribution to the understanding of performance sensitive bank debt. In addition to information asymmetry and agency costs, they claim that the existence of renegotiation costs provides another rationale for using PSD contracts. The authors find empirical evidence that performance sensitive debt is used when it has the largest net benefits, i.e., when moral hazard, adverse selection problems, or renegotiation costs are likely to be high. More specifically they find that interest decreasing

PSD contracts are used when prepayment of the loan is more likely, i.e., when borrowers' relative bargaining power is assumed to be high and when costs related to adverse selection are large. They also find that interest increasing PSD contracts are used when moral hazard costs are high and that including interest increasing provisions in the debt contract has significant economic effects since, controlling for firm characteristics, borrowers are offered 26 basis points initial lower credit spreads (over LIBOR) when these provisions are included in the contract.

Other important contributions to the PSD literature includes Roberts & Sufi (2009) on debt renegotiations and Tchistyi, Yermack, & Yun (2010) on CEO equity incentives.

Several papers empirically test different structural models of credit risk, focusing on the models' ability to replicate observed market prices and credit spreads. The evidence is mixed. Jones, Mason, & Rosenfeld (1984) find that predicted prices are, on average, 4.5% too high, and that the pricing error is largest for speculative-grade firms. More recently, Eom et al. (2004) compare five different models, and find that predicted credit spreads from some are too high, whereas some models generate too low credit spreads. Huang & Huang (2003) also test several different models. They use a calibration approach based on historical data, and find that credit risk accounts for only a small fraction of observed corporate credit spreads for investment grade bonds, but accounts for a larger share of high-yield bond credit spreads. They also find that different structural models predict fairly similar credit spreads.

2 The economic model

A General set-up

This section reviews the general set-up and the main results needed for our pricing model. A filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$ is given. In particular, Q represents a fixed equivalent martingale measure. We impose the standard frictionless, continuous time market assumptions, see, e.g., Duffie (2001).

To implement shocks to the borrower's credit quality we include jumps in the amount of debt. A jump in the amount of debt is a tractable example of a shock to borrower credit quality. Even though we let debt follow a jump process, we disregard any overall capital structure considerations in our analysis.

Let W_t be a standard Brownian motion under the equivalent martingale measure Q. We assume throughout that the time t firm cash flow rate ζ_t under the equivalent martingale measure Q is given by the stochastic differential equation

$$d\zeta_t = \mu\zeta_t dt + \sigma\zeta_t dW_t, \tag{1}$$

where the initial value ζ_0 is a constant. Here the drift parameter μ and the diffusion parameter σ are constants. Denoting the constant risk-free interest rate by r, where $r > \mu$, the time t value of the firm's assets A_t equals the risk-adjusted expected discounted value of all future cash flows

$$A_t = E_t^Q \left[\int_t^{\infty} e^{-r(s-t)} \zeta_s ds \right] = \frac{\zeta_t}{r-\mu}, \tag{2}$$

where $E_t^Q[\cdot]$ denotes the expectation under the equivalent martingale measure Q conditional on $\mathcal{F}_t$, the information available at time t. Hence, the market value of the firm's assets is given by expression (3) divided by the constant $(r-\mu)$. In particular, $A_0 = \frac{\zeta_0}{r-\mu}$.

Let N_t denote a Poisson process with constant intensity λ under the equivalent martingale measure Q. Let $\{Y_i\}, i \geq 1$, be a sequence of independent and identically distributed random variables under Q, independent of both N_t and W_t. We assume that the firm is partly financed by debt and that the total book value of debt at time t is

$$D_t = D_0 \prod_{i=1}^{N_t} Y_i,$$

where $D_t = D_0$ when $N_t = 0$. Here Y_i can be interpreted as the change in debt due to jump i relative to the amount of debt just before jump i. Realized values of $Y_i > 1$ implies an increase in the amount of debt, and realized values of $Y_i < 1$ implies a decrease in the amount of debt.

The state variable in our model is the ratio between the cash flow and debt. Define ξ_t by

$$\xi_t = \frac{\zeta_t}{D_t} = \xi \exp\left((\mu - \frac{1}{2}\sigma^2)t + \sigma W_t \right) \prod_{i=1}^{N_t} (Y_i)^{-1}, \tag{3}$$

where the initial value $\xi = \xi_0 = \frac{\zeta_0}{D_0}$ is a constant.

Let T be the finite time horizon corresponding to the maturity of debt. Let the constant $C < \xi$ be an absorbing barrier, and define the stopping time τ (with respect to $\mathcal{F}_t$) as

$$\tau = \inf\{t \geq 0, \xi_t \leq C\}. \tag{4}$$

The constant C can be interpreted as the contractual default barrier, and τ as the time of default.

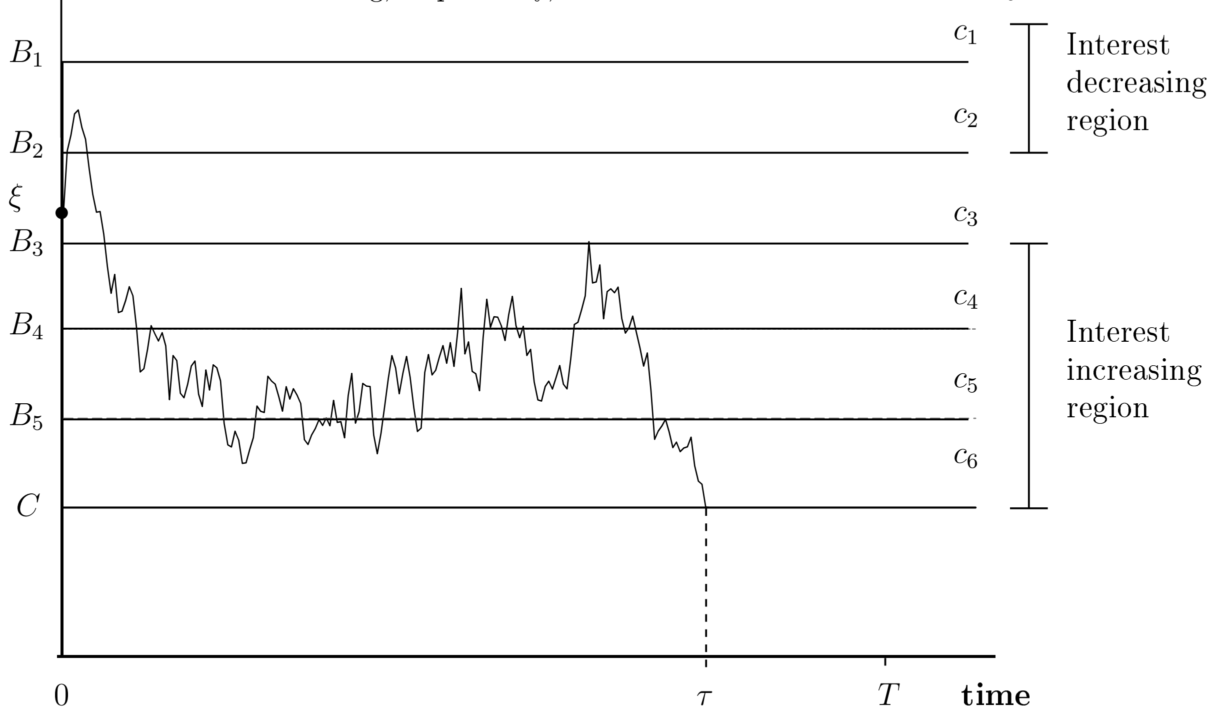
B A Valuation Model of a PSD Contract

This subsection explains the general structure of a PSD contract. In addition to the contractual default barrier C, a PSD contract includes $n+m$ constant levels or non-absorbing barriers $B_1, \ldots, B_{n+m}$ so that $B_1 > \cdots > B_{n+m} > C$. For notational simplicity only, we let $B_0 = \infty$ and $B_{n+m+1} = C$. We define n by the initial value of the credit performance measure ξ as $B_n > \xi > B_{n+1}$. That is, there are n barriers above ξ. Similarly, m represents the number of barriers below ξ. Observe that the contract is well defined in the cases where $n = 0$ and/or $m = 0$.

The contract specifies a sequence of interest rates, where c_{i+1} is paid when $B_i > \xi_t > B_{i+1}$, $i = 0, ..., n + m$. All c_is are assumed to be constants. An interest increasing contract is defined by $n = 0$ and $c_1 < c_2 < ... < c_m$, whereas an interest decreasing contract is defined by $m = 0$ and $c_1 < c_2 < ... < c_n$. See Figure 1 for an illustration.

Illustration of a PSD Contract

Figure 1: The graph contains an example of a path of the credit performance measure ξ_t and indicates in which regions the interest rates are c_1,..., c_6 respectively. Also, ξ, B_1,...,B_5, C, T, and τ are depicted. C denotes the contractual default barrier. The number of non-absorbing barriers above the starting level ξ is $n = 2$, and the number of non-absorbing barriers below ξ is $m = 3$. In order to make the two indicated regions interest decreasing and interest increasing, respectively, it is assumed that $c_1 < c_2 < ... < c_6$.



The total time 0 market value of a PSD contract can be decomposed into the time 0 market value of the interest payments and the time 0 market value of the repayment of the principal. Let us first consider the market value of the interest payments. Consider the corridor j defined by two adjacent barriers B_j and B_{j+1} for a fixed j. In this corridor the interest rate is c_{j+1}. The market value of interest payments in a time period $[0, T]$ from this corridor is

$$C_j(\xi) = E^Q\left[\int_0^{\tau\wedge T} c_{j+1}e^{-rs}1\{B_j > \xi_s > B_{j+1}\}ds\right],$$

where $E^Q[\cdot]$ denotes the expectation under the equivalent martingale measure Q. The total interest payments of the PSD contract can be seen as a portfolio of such corridors. To calculate the total time 0 market value of all interest payments from a PSD contract, we add the time 0 market values of the contract's corridors, i.e., $V(\xi) = \sum_{i=0}^{n+m} C_i(\xi)$ or

$$V(\xi) = E^Q\left[\int_0^{\tau\wedge T} \sum_{i=0}^{n+m} c_{i+1}e^{-rs}1\{B_i > \xi_s > B_{i+1}\}ds\right]. \tag{5}$$

The corridor decomposition in expression (5) is the basis for the subsequent simulation analysis of the interest payments in our sample of contracts.

We find it useful to further decompose a corridor by the use of above- and below annuities, see Mjøs & Persson (2010). A generic defaultable finite horizon *above* annuity pays the annuity rate of 1 when the credit performance measure is above some level B until default or to the horizon, whatever comes first. Denote the time 0 market value of an above annuity by $\mathcal{A}(\xi, B)$, then

$$\mathcal{A}(\xi, B) = E^Q\left[\int_0^{\tau\wedge T} e^{-rs}1\{\xi_s > B\}ds\right].$$

A generic defaultable finite horizon *below* annuity pays the annuity rate of 1 when the credit performance measure is below some level B until default or to the horizon, whatever comes first. Denote the time 0 market value of a below annuity by $\mathcal{B}(\xi, B)$, so

$$\mathcal{B}(\xi, B) = E^Q\left[\int_0^{\tau\wedge T} e^{-rs}1\{\xi_s < B\}ds\right].$$

Observe that for any B, $\mathcal{A}(\xi, B) + \mathcal{B}(\xi, B) = Z(\xi)$, where $Z(\xi)$ is the time 0 market price of finite horizon interest payments with rate identical to 1, i.e.,

$$Z(\xi) = E^Q\left[\int_0^{\tau\wedge T} e^{-rs}ds\right].$$

A corridor j can be decomposed in two equivalent ways. First, as a portfolio of a long defaultable above annuity with annuity payment c_{j+1} and level B_{j+1}, and a short defaultable above annuity with annuity payment c_{j+1} and level B_j. Second, as a portfolio of a long defaultable below annuity with annuity payment c_{j+1} and level B_j, and a short defaultable below annuity with annuity payment c_{j+1} and level B_{j+1}. The time 0 market value of corridor j, using above annuities, is

$$C_j(\xi) = (\mathcal{A}(\xi, B_{j+1}) - \mathcal{A}(\xi, B_j))c_{j+1}.$$

The time 0 market value of corridor j, using below annuities, is

$$C_j(\xi) = (\mathcal{B}(\xi, B_j) - \mathcal{B}(\xi, B_{j+1}))c_{j+1}.$$

Using above and below annuities, the total time 0 market value of all interest payments from a PSD contract can be expressed as

$$V(\xi) = c_{n+m+1} Z(\xi) - \sum_{i=1}^{m+n} \mathcal{A}(\xi, B_i)(c_{i+1} - c_i), \tag{6}$$

or

$$V(\xi) = c_1 Z(\xi) + \sum_{i=1}^{m+n} \mathcal{B}(\xi, B_i)(c_{i+1} - c_i), \tag{7}$$

Proof. Both these expressions follow from the relationship $V(\xi) = \sum_{i=0}^{n+m} C_i(\xi)$ and the above definitions of corridors, using above- and below annuities, respectively. Observe that $\mathcal{A}(\xi, B_0) = 0, \mathcal{A}(\xi, B_{m+n+1}) = Z(\xi), \mathcal{B}(\xi, B_0) = Z(\xi), \mathcal{B}(\xi, B_{n+m+1}) = 0.$ $\square$

These expressions may be interpreted in the following ways. Expression (6) suggests that, in principle, the borrower pays the highest interest rate c_{n+m+1} throughout the term of the contract, but for each (additional) barrier away from the default level the credit performance measure process is, the borrower is entitled to a lower interest rate. The interest rate discount is determined by the interest rate difference between each barrier and the time 0 market price of an above annuity. Alternatively, expression (7) suggests that, in principle, the borrower pays the lowest interest rate c_1 throughout the term of the contract, but for each barrier closer to the default level the credit performance measure process is, the borrower has to pay an increased interest rate. The additional interest rate is determined by the interest difference between each barrier and the time 0 market price of an below annuity. Although either of these interpretations may be used for any PSD contract of the type we consider, two special cases are worth emphasizing. An *interest decreasing* contract is characterized by the fact that the initial interest rate of the contract equals the highest possible interest rate. The first interpretation above, based on expression (6) seems more intuitive for this contract. An *interest increasing* contract is characterized by the fact that the initial interest rate of the contract equals the lowest possible interest rate. The second interpretation above, based on expression (7) seems more intuitive for this contract.

We now turn to the time 0 value of the repayment of the principal. First, let $Q(\xi) = Q(\tau > T)$ be the survival probability under the equivalent martingale measure Q. The time 0 market value of receiving the face value of debt (D) in the case of no default is, thus, $De^{-rT}Q(\xi)$. The time 0 market value of the recovery amount in the case of default is, similarly, $D(1-\kappa)H(\xi)$. The parameter κ represents the debtholders' loss proportional to the face value of debt in case of contractual default. Here, $H(\xi) = E^Q[e^{-r\tau}1\{\tau \leq T\}]$ represents the time 0 value of one unit of currency paid upon default if default occurs before time T. The total time 0 value of a PSD contract is, thus,

$$L(\xi) = V(\xi) + De^{-rT}Q(\xi) + D(1-\kappa)H(\xi). \tag{8}$$

C PSD Valuation Assuming No Shocks to Credit Quality

In the special case of no jumps in the debt level D_t, i.e., $N_t \equiv 0$ and $\lambda = 0$, we can calculate a closed form formula for the present value of a PSD contract. In the following we use the superscript c to denote closed form solutions. The closed form expression for $Q^c(\xi)$ and $H^c(\xi)$ are standard and can be found in Appendix A. In order to calculate $V(\xi)$ in the case of no jumps, our starting point is expression (6) or expression (7). Mjøs & Persson (2010) calculate closed form solutions for $\mathcal{A}(\xi, B)$ and $\mathcal{B}(\xi, B)$. These formulas depend on whether the initial value of the credit performance measure ξ is above or below the barrier, B. We therefore write

$$V^c(\xi) = c_{n+m+1}Z^c(\xi) - \sum_{i=1}^{n} \mathcal{A}_b^c(\xi, B_i)(c_{i+1} - c_i) - \sum_{i=n+1}^{n+m} \mathcal{A}_a^c(\xi, B_i)(c_{i+1} - c_i), \tag{9}$$

where $\mathcal{A}_b^c(\xi, B_i)$ is the time 0 market value of an above annuity with barrier B_i where $\xi < B_i$, and $\mathcal{A}_a^c(\xi, B_i)$ is the time 0 market value of an above annuity with barrier B_i where $\xi > B_i$. The expressions for $\mathcal{A}_b^c(\xi, B_i)$ and $\mathcal{A}_a^c(\xi, B_i)$ are given in expressions (28) and (24) in Appendix A. The expression for $Z^c(\xi)$ is given in expression (21) in Appendix A. The similar expression based on below annuities is

$$V^c(\xi) = c_1 Z^c(\xi) + \sum_{i=1}^{n} \mathcal{B}_b^c(\xi, B_i)(c_{i+1} - c_i) + \sum_{i=n+1}^{n+m} \mathcal{B}_a^c(\xi, B_i)(c_{i+1} - c_i), \tag{10}$$

where $\mathcal{B}_b^c(\xi, B_i)$ is the time 0 market value of a below annuity with barrier B_i where $\xi < B_i$, and $\mathcal{B}_a^c(\xi, B_i)$ is the time 0 market value of a below annuity with barrier B_i where $\xi > B_i$. The expressions for $\mathcal{B}_b^c(\xi, B_i)$ and $\mathcal{B}_a^c(\xi, B_i)$ are given in expressions (31) and (29) in Appendix A. The closed form formula for the total value of the contract in the case of no jumps is, thus, given by

$$L^c(\xi) = V^c(\xi) + De^{-rT}Q^c(\xi) + D(1-\kappa)H^c(\xi), \tag{11}$$

where $V^c(\xi)$ is given either by expression (9) or expression (10). Furthermore, $Q^c(\xi)$ and $H^c(\xi)$ are given by expressions (17), and (20), respectively.

3 Market and Data Description

This section gives an overview of the market for PSD contracts and describes the data we use to empirically test our pricing model.

A Overview and Descriptive Statistics

The tables and statistics in this section describe the PSD contracts in the Thomson Reuter's Dealscan database. We have collected all available data for the years 1993 - 2010, resulting in a total of 218,204 loans. The database contains 218,204 loans for the years 1993 - 2010, with detailed information about the global commercial loan market, focusing primarily on corporate bank debt with longer maturities. The database provides information for both publicly traded and privately held debt[2]. The PSD part of the database includes 25,602 loans. The total outstanding principal of these PSD contracts is USD 9,900 bn. (25.6% of the total outstanding amount). One deal may consist of several loans, usually referred to as tranches. The data in Dealscan is reported by loans and, thus, all our data analysis is at loan level rather than deal level. Table I reports the use of different types of credit

[2]See Carey et al. (1998) for a more detailed description of the database.

performance measures in PSD contracts. Total debt-to-cashflow and senior debt rating are the two most common credit performance measures in these contracts. In total, 51.3% of the PSD contracts are directly related to cash-flow (25.4% of loan amount), and could potentially be valued using our model.

PSD Credit Performance Measures

Table I: This table shows the numbers of loans with different types of credit performance measures as a percentage of the total number of loans containing performance pricing provisions (N=25,602 loans), and as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

Performance measure	Total number of loans	Total loan amount
Total debt-to-cashflow	47.6 %	23.2 %
Senior debt rating	25.8 %	53.5 %
Leverage	5.7 %	3.9 %
Maturity	4.2 %	5.7 %
Senior debt-to-cash flow	3.7 %	2.2 %
Outstandings	2.0 %	3.4 %
Fixed charge coverage	2.3 %	0.7 %
Debt to tangible net worth	2.0 %	0.6 %
Interest Coverage	2.5 %	1.6 %
Debt Service Coverage Ratio	0.8 %	0.1 %
Other	3.4 %	5.1 %

Table II shows the distribution of such debt contracts according to some broadly defined financing purposes. PSD contracts specify a verifiable credit performance measure in order to trigger changes in interest rates. This fact may explain why such contracts are primarily used by firms for which audited financial reports are available. PSD contracts are seldom used for project financing. Table III shows the existence of a credit rating for 50.3% of PSD-borrowers compared to 13.7 % of other borrowers. Of all PSD-borrowers, 29.8 % are rated below investment grade compared to 7.7 % of other borrowers. In addition, we find that

Purpose of PSD

Table II: This table shows the purpose of performance sensitive loans as a percentage of the total number of issued loans containing performance sensitive provisions (N=25,602), as well as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

Purpose	Total number of loans	Total loan amount
Acquisition-related	21.5 %	26.7 %
Refinancing	23.7 %	16.9 %
Working Capital	18.5 %	11.5 %
Project Finance	1.6 %	1.3 %
All Other	34.7 %	52.0 %

with negligible exceptions, all performance sensitive loans are senior (99.9%). Also, 57.7% of the loans are secured, whereas 23.7% are unsecured (information regarding security is not available for the remaining 20%.). Table IV shows that USA and Canada alone account

Borrower Ratings

Table III: This table shows the distribution of initial borrower ratings (S&P/Moody's senior debt ratings respectively) for both PSD (N=25,602) and non-PSD loans (N=192,602). The numbers are calculated as the number of loans with a given borrower credit rating divided by the total number of loans for the given category. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

Rating Category	PSD	Non-PSD
AAA/Aaa	0.3 %	0.4%
AA/Aa	0.9 %	0.7%
A/A	5.8 %	2.2%
BBB/Baa	13.5 %	2.7%
BB/Ba	13.6 %	3.1%
B/B	13.2 %	4.1%
CCC/Caa	1.0 %	0.5%
CC/Ca	0.0 %	0.0%
C /C	0.0 %	0.0%
Sum Investment Grade	20.5 %	6.0 %
Sum Non-Investment Grade	29.8 %	7.7 %
Not Rated	49.7 %	86.3%

for almost 90% of these contracts which may be explained by the historically high level of sophistication of the financial markets in this region. Market participants indicate that more than half of recently issued syndicated bank loans in Europe include such provisions.

Geographical Distribution of PSD

Table IV: This table shows the geographical distribution of performance sensitive loans as an equal-weighted percentage of the total number of such loans (N=25,602). Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

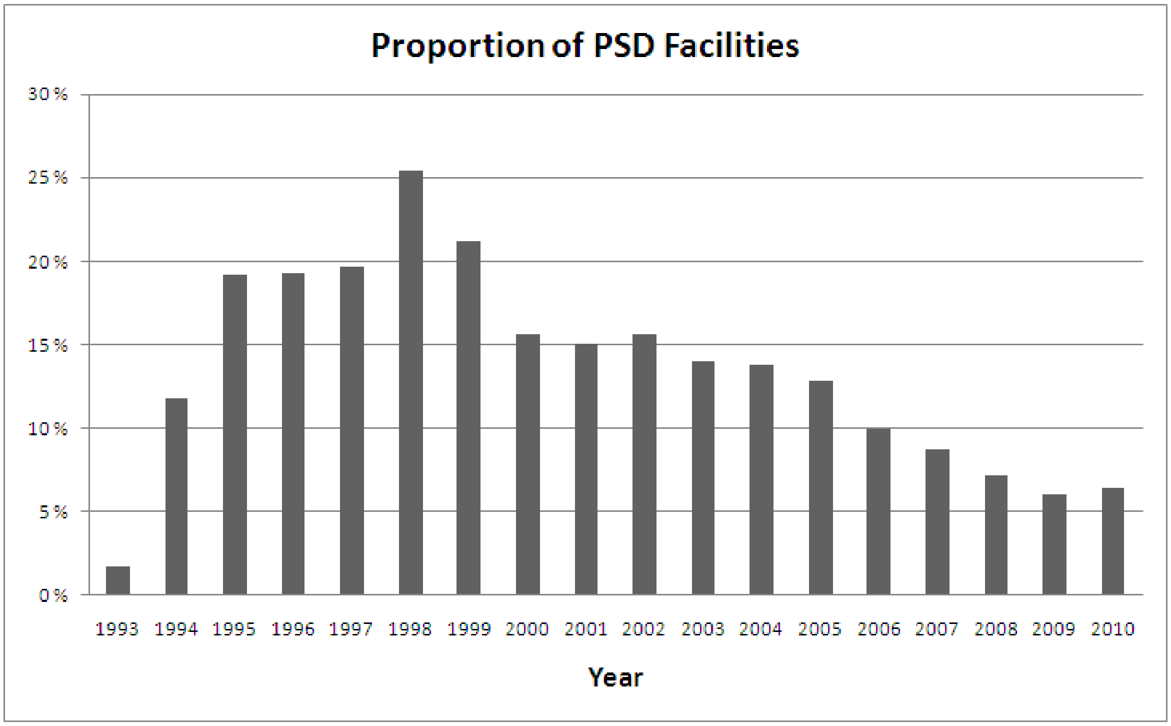
Borrower region	**Total number of loans**
USA/Canada	88.8 %
Western Europe	6.5 %
Latin America/Caribbean	1.7 %
Asia-Pacific	1.9 %
Eastern Europe/Russia	0.6 %
Middle East	0.4 %
Africa	0.2 %

Figure 2 shows the use of performance sensitive loans, relative to all new loans during the last 17 years. We note that a substantial fraction of loans include performance sensitive provisions.

Appendix B includes tables showing descriptive statistics for maturity and loan amounts comparing performance sensitive debt and non-PSD debt, as well as an overview of broad borrower industry classes. Loan amounts are larger for PSD contracts than for regular loans, but the maturities do not differ between the categories. PSD contracts are divided appr. 40/60 between term loans and revolvers. We disregard any commitment fees and since actual drawdown behavior of revolvers do not influence their initial pricing , our analysis of pricing is therefore equally applicable to both categories.

Relative Use of PSD

Figure 2: The histogram shows the annual proportion of performance sensitive loans relative to the total number of new loans. Numbers are based on data from Thomson Reuter's Dealscan database for the years 1993-2010.



B Data Description

B.1 Sample Construction

Our sample is extracted from Thomson Reuter's Dealscan[3] in March 2011. We collect all loans with performance pricing provisions, a total of 27,994 loans issued in the period 1993 - 2010. We confine our analysis to contracts with interest rates linked to the company's debt-to-cash-flow ratio (Debt/CF), a condition for the valuation model in Section 3. This restriction reduces the number of loans to 8,180. In addition to the performance sensitive feature, we require the existence of a debt-to-cash-flow default covenant in the contract. This

[3]See www.loanpricing.com for more information on the database and how to access it.

requirement reduces our sample to 6,727 loans. We further restrict our sample to publicly listed borrowers with sufficient market and company information from the databases CRSP and Compustat prior to the inception of the loan. Information from these databases is used to estimate the drift and volatility parameters of the borrowers' cash flow process in expression (1). Hence, we require the borrowing company to be listed in the two latter databases, and to have a minimum of 1 year of historical data for parameter estimation[4]. This last restriction reduces our sample to 5,143 loans. To ensure compatibility with our model, we also remove all contracts where the estimated starting value of the Debt/CF process is below the default covenant, the loan has no stated maturity, the number of barriers do not match the number of different loan credit spreads, or the loan credit spreads are not varying across different barriers. Some of the remaining loans in our sample are identical and come from the same loan deal. To avoid duplications, we keep only one loan from each deal. Our final sample consists of 3,052 loans. The sample includes 342 interest decreasing contracts, 1,520 interest increasing contracts, and 1,190 contracts containing both categories of performance sensitive interest rates. All loans are senior and secured, and are granted by banks in the time period 1993 - 2010. 93% of all loans are granted to U.S. firms, whereas the remaining 7% are granted to European or Canadian firms.

[4] To ensure correct matching of companies in Dealscan with Compustat/CRSP we use the Dealscan-Compustat links from Chava & Roberts (2008). We thank Michael Roberts for providing us with the matching file.

B.2 Sample Presentation

Table V lists summary statistics of our sample. The loans have from 1 - 8 non-absorbing Debt/CF barriers with a mean of 3.46 and a median of 3. The size of the loans varies from USD 0.6m to USD 10.7bn, whilst the average loan amount is USD 181m. No PSD loan has maturity above 21 years, whilst the average maturity is 4.54 years. The average initial credit spread, measured by the all-in-spread (AIS), is 187 basis points, with a sample standard deviation of 86 basis points. When we analyze firms using PSD loans we find no particular size distribution. The average borrower profitability, measured by the quarterly return on capital employed (ROCE), is 4.1%, with a median of 3.5% and a standard deviation of 4.9%. The average initial market leverage, defined as the book value of debt divided by the by the sum of book value of debt and market value of equity, just prior to establishing the PSD loan, is 0.27. The corresponding median and standard deviation are 0.23 and 0.20, respectively. To measure the relative significance of the PSD loan in the borrowers' total leverage, we estimate the ratio of the PSD loan divided by the total debt (the sum of existing debt and the new PSD loan). The average share of the new PSD loan relative to the borrower's total debt is 48%, with a median of 46% and a standard deviation of 26%. We do not know whether the borrowers' existing debt has PSD provisions. In our sample, 46% of the borrowers are rated, and 37% of the rated borrowers are rated investment grade[5]. The average and median of the sample estimated annual cash flow volatility are 11.9% and 4.4%, respectively. These volatility estimates range from 2% to 80%. The average distance-

[5]This is the general senior Standard & Poor's rating of the borrower's creditworthiness with respect to its long-term financial obligations such as senior debt. The bank loans we analyze are not rated separately.

to-default (DTD), defined as the starting value of the CF/Debt measure less the contractual default barrier, and normalized by the cash flow volatility, is 49.2 with a standard deviation of 114.8. In Table V we include the DTD characteristics for interest increasing and interest decreasing loans, respectively. Table V shows, as we would expect, that interest increasing contracts have a larger DTD compared to interest decreasing contracts. Summary statistics for each of the three subsamples (interest increasing, interest decreasing, both provisions) can be found in Tables XVI, XVII, and XVIII in Appendix B. See also Tables XII and XIII for a statistical analysis by loan category. These tables show that borrowers using interest increasing PSD contracts are more profitable and less levered compared to borrowers using interest decreasing PSD contracts. Furthermore, they have a lower cash flow volatility and a larger contractual DTD. Firms using interest increasing PSD contracts also pay a lower initial credit spread compared to borrowers using interest decreasing PSD contracts. These observations unanimously suggest that borrowers using interest increasing PSD loans are of an overall higher credit quality than those using interest decreasing PSD loans, when the loan is granted.

To assess how representative our sample is, we compare it to the population of PSD loans in the database. First of all, note that the sample borrower ratings correspond well with the observations in Table III. For the entire database (our sample means in brackets) the average maturity is 4.5 (4.54) years, the average loan amount is USD 369m (USD 181m), the average borrower's quarterly sales volume is USD 2,658m (USD 986m) and the average

AIS is 194.6 (187) bp. These statistics, see Table VI, suggest that our sample consists of somewhat smaller loans as well as smaller borrowers, but that the initial loan credit spreads are close to the average of the population of PSD loans.

Full Sample Summary Statistics

Table V: This table shows summary statistics for various model input parameters and borrower characteristics for the final sample used in the paper. The loan contracts in the sample are granted in the period 1993-2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics						
Company Sales (MUSD)	986	424	3,321	1.1	159,098	3,052
ROCE (%,quarterly)	4.09	3.53	4.94	-17.09	112.17	3,052
Leverage (Debt/(Debt + Equity))	0.27	0.23	0.20	0.00	1.00	3,052
PSD Loan/Total Debt	0.48	0.46	0.26	0.003	1.00	3,052
Drift of cash flow ($r - \delta$)	0.023	0.025	0.014	-0.09	0.06	3,052
Volatility of cash flow (σ)	0.12	0.044	0.171	0.02	0.80	3,052
Loan Characteristics						
Loan Amount (MUSD)	181	100	316	0.6	10,700	3,052
Loan Maturity (Years)	4.54	5.00	1.68	0.08	21	3,052
All-In-Spread (Bp)	187	175	86	23	750	3,052
# of Barriers	3.46	3	1.25	1	8	3,052
Distance-to-default (DTD)						
- Full Sample	49.2	6.6	114.8	0.00	1092.6	3,052
- Interest increasing	94.0	28.3	149.7	0.03	1092.6	1,520
- Interest decreasing	1.2	0.34	2.15	0.00	24.8	342

Comparison of PSD Sample and PSD Population

Table VI: Table shows sample averages as well as population averages for available variables (Borrowers' sales, loan maturity, loan amount and All-in-spread). Population is defined as all PSD loans in Thomson Reuter's Dealscan database in the period 1993 - 2010.

	PSD Sample			PSD Population		
Variable	**Mean**	**Median**	**Std. Dev.**	**Mean**	**Median**	**Std. Dev.**
Company Sales (MUSD)	986	424	3,321	2,658	581	9,866
Loan Maturity (Years)	4.54	5.0	1.68	4.5	5.8	1.94
Loan Amount (MUSD)	181	100	316	369	149	879
All-In-Spread (Basis points)	187.5	175.0	86.5	194.6	175.0	116.5
N		3,052			27,994	

Terms of Example Contract

Table VII: This table provides an overview of the main terms in the chosen example PSD contract.

Borrower	**Actuant Corp.**
Deal Active Date	31 Jul 2000
Amount	USD 100 m
Loan type	Term Loan
Seniority	Senior
Maturity	72 months
Distribution Method	Syndication
Lead Bank	Credit Suisse First Boston
Reference Rate	LIBOR 3 mth
Type of pricing grid	Interest decreasing
Initial CF/Debt ratio	0.24
Borrower Senior Debt Rating (S&P)	BB

4 Pricing a PSD Contract in the Case of No Shocks to Credit Quality

In order to show how the model input parameters are estimated and how these parameters influence pricing, we select the first loan contract in our sample (sorted alphabetically by borrower name), and illustrate the pricing of this contract using our closed form pricing formula (11). The results for the entire sample are included and analyzed in the next section.

A Pricing of an Example Contract - No Shocks to Credit Quality

Actuant Corporation[6] borrowed USD 100m in the year 2000 using a PSD contract. The main terms of this contract are given in Table VII. The credit performance measure in this contract links the interest paid on the loan to the performance of the company via the com-

[6]See www.actuant.com for more information on the company.

Interest Rate Grid of the Example Contract

Table VIII: Table shows how the interest rate is linked to company cash flow through the Debt/CF ratio and the CF/Debt ratio respectively. A Debt/CF ratio equal to 4.55, or equivalently a CF/debt ratio of 0.22, represents the maximum (minimum) ratio that is accepted by the contract terms, and hence is the exogenously given contractual default barrier.

		Performance Measure			**Interest Margins**	
Ranges	**Barriers**	**Debt/CF**	**CF/Debt**	**LIBOR Spread**	**Commitment Fee**	**Total Spread**
1	(B_1, B_0)	$[0, 1.75)$	$(0.57, \infty)$	150	25	175
2	$(B_2, B_1]$	$[1.75, 2)$	$(0.5, 0.57]$	162.5	37.5	200
3	$(B_3, B_2]$	$[2, 2.5)$	$(0.4, 0.5]$	187.5	37.5	225
4	$(B_4, B_3]$	$[2.5, 3)$	$(0.33, 0.4]$	212.5	37.5	250
5	$(B_5, B_4]$	$[3, 3.5)$	$(0.29, 0.33]$	225	50	275
6	$(B_6, B_5]$	$[3.5, 4.25)$	$(0.24, 0.29]$	250	50	300
7	$(\mathbf{C}, B_6]$	$[4.25, \mathbf{4.55})$	$(\mathbf{0.22}, 0.24]$	275	50	325

pany's Debt/CF ratio as shown in Table VIII[7]. Debt equals the book value of total debt, and is found by adding the company's long-term debt and debt in current liabilities (DLTTQ + DLCQ). We invert the Debt/CF ratio and assume that the borrower has a constant total debt level until the maturity of the loan. Also note that this is an *interest decreasing* contract since the starting level of the inverted performance measure (CF/debt) is 0.24, i.e., at the lowest non-absorbing barrier. In order to price this contract we estimate the drift and the volatility of the underlying cash flow process. To estimate the drift we use the insight of Goldstein et al. (2001) that the growth of the cash flow process, under the equivalent martingale measure Q, equals the risk-free rate r if the company retains all of its earnings. However, a company with a payout rate δ proportional to earnings, has a lower drift under Q equal to $r - \delta$. Payouts to investors and government typically consist of dividends to shareholders,

[7] The USDLIBOR (London Interbank Offered Rate) spread indicates the contractual spread over LIBOR 3 mth USD rate measured in basispoints. The total spread equals the sum of the LIBOR spread and a commitment fee. CF equals cash flow and is proxied in the loan contract, and in our analysis, by reported EBITDA (Earnings Before Interest, Tax, Depreciations and Amortizations).

interest payments to debtholders, and tax payments to the government. Collecting quarterly data (Compustat codes in brackets) on EBITDA (OIBDQ), interest expenses (XINTQ), income taxes (TXTQ) and total dividends (DVTQ) from Compustat, we estimate δ by the ratio (XINTQ + TXTQ + DVTQ)/OIBDQ. The data is collected from the fiscal quarter before the loan is granted to ensure that the data would be common information available to all parties[8]. For Actuant Corp. the estimate is $\delta = 0.0218$.

The volatility of the cash flow process is not an observable input parameter in our model. From equations (3) and (2) we know that the volatility of the cash flow is equal to the volatility of the firm's assets. In order to estimate the asset volatility we adopt the procedure used in Vassalou & Xing (2004) and Bharath & Shumway (2008). This procedure utilizes the insights from Merton (1974) that equity is a call option on a firm's assets, where the strike price of the call option is the face value of the firm's debt. The expiration date of the option corresponds to the maturity of the debt. Recall that A_t and D_t denote the market value of assets and the book value of debt, respectively. Define the value of equity at time t by E_t. Using the Black-Scholes formula the time t market value of equity is given by

$$E_t = A_t N(d_1) - D_t e^{-r(T-t)} N(d_2), \tag{12}$$

[8]One could alternatively use a longer time series to estimate δ. As a robustness check we estimate δ for all borrowers in our sample, using information from the last 4 quarters prior to loan inception. This procedure tend to produce slightly higher payout rates, but the valuation effects are negligible.

where

$$d_1 = \frac{\ln\left(\frac{A_t}{D_t}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t}. \tag{13}$$

We estimate σ using the following iterative procedure. We download daily stock price data from the past 12 months prior to the loan inception, calculate the volatility of equity σ_E, and use this as our initial guess for the estimation of σ. Using expression (12) we compute A_t for each trading day for the past 12 months, using the observed market value of equity of that particular day, and the last known observation of D_t. Thus, we create a new daily time series of A_t, and estimate σ from this new time series. This estimate is then used as input for the next iteration. This procedure is repeated until the values of σ from two consecutive iterations converge. The tolerance level for convergence is 0.0001. Following Bharath & Shumway (2008) we set $T = 1$ and define the face value of debt D_t as debt in current liabilities (Compustat item DLCQ) plus one half of long-term debt (Compustat item DLTTQ). In the case of Actuant Corp our estimate of the annual asset volatility, equal to the annual cash flow volatility, is $\sigma = 0.1188$.

As an approximation for the risk-free rate we use the quote of the 3-month T-bill rate in the month prior to the the loan issuance date[9], i.e., in this case June 2000. This risk-free rate equals 5.83%, and implies that $\mu = r - \hat{\delta} = 0.0583 - 0.0218 = 0.0365$. The USD LIBOR 3 month rate is used as the reference interest rate in all loan contracts in our sample. To

[9]These are collected from the Federal Reserve's official statistical releases (http://www.federalreserve.gov/releases/h15/data.htm). The website also contains descriptions of how these rates are measured.

find the correct interest rates to use throughout the loan period as our model input, we add the contractual credit spreads to the forward LIBOR rates. Forward LIBOR rates are only available up to one year maturity, and hence we proxy longer-term forward rates by swap rates obtained from the quoted swap-curve when the loan is granted[10]. In this example, the maturity of the loan is 6 years, and we use the 6 year forward swap rate quoted in June 2000 as our reference rate. Adding the contractually determined credit spreads yields the interest rates. As the contractual default barrier C, we use the financial covenant stating that the maximum Debt/EBITDA ratio should not be above 4.55. This ratio corresponds to a value of C (i.e., CF/Debt) equal to $1/4.55 = 0.22$ in our model. The starting value of the asset process, i.e., the current value of CF/Debt is 0.24. As an approximation of the recovery rate ($1 - \kappa$) we use the estimated recovery rate for senior secured bank debt from Altman, Resti & Sironi (2004). This recovery rate[11] is 73%, implying that the liquidation cost parameter κ equals 27%. The liquidation cost parameter determines the loss in the case of default. The size of the loss depends on whether the default leads to a full liquidation or not. Based on the explanation in the introduction regarding the presence of cross-defaults, we choose to apply the estimated liquidation cost parameter of 27%, even if we value a single debt contract and not necessarily a borrower's total debt. Practitioners confirm the magnitude of this parameter. The face (par) value of debt is normalized to 100. Table IX summarizes the values of our input parameters.

[10] See also www.federalreserve.gov/releases/h15/data.htm for information on swap curves.
[11] The estimated recovery rate is based on recovery of principal 30 days after default.

Numerical Values of Input Parameters for the Example Contract

Table IX: This table states the value of all relevant input parameters needed to estimate the price of the example PSD contract, as described in Tables VII and VIII.

Parameters	Values	Explanations
T	6	Maturity, in years
D	100	Face value of debt, normalized
B_1	0.57	Barrier 1 (CF/Debt)
B_2	0.50	Barrier 2 (CF/Debt)
B_3	0.40	Barrier 3 (CF/Debt)
B_4	0.33	Barrier 4 (CF/Debt)
B_5	0.29	Barrier 5 (CF/Debt)
B_6	0.24	Barrier 6 (CF/Debt)
C	0.22	Default Barrier (CF/Debt)
c_1	0.0892	Interest rate paid when $A_t \geq B_1$
c_2	0.0917	Interest rate paid when $B_1 > A_t \geq B_2$
c_3	0.0942	Interest rate paid when $B_2 > A_t \geq B_3$
c_4	0.0967	Interest rate paid when $B_3 > A_t \geq B_4$
c_5	0.0992	Interest rate paid when $B_4 > A_t \geq B_5$
c_6	0.1017	Interest rate paid when $B_5 > A_t \geq B_6$
c_7	0.1042	Interest rate paid when $B_6 > A_t > C$
A	0.24	Starting value of the CF/Debt process
μ	0.0365	Risk-neutral drift of the CF/Debt process
σ	0.1188	Volatility of the CF/Debt process
r	0.0583	Risk-free interest rate
κ	0.27	Liquidation cost parameter

Given the parameter values in Table IX the market value of the PSD example contract at issue is 95.44, calculated using expression (11). Thus, the theoretical market price is below the par value of 100, and this contract is underpriced.

B Decomposition of Interest Increasing and Interest Decreasing PSD Contracts

In order to interpret our empirical results we find it useful to decompose a PSD contract into a sum of a fixed rate loan and an option portfolio. From a lender's point of view an *interest increasing* PSD contract is equivalent to a fixed rate loan plus a portfolio of long put options. The put options give the lender rights to receive increased interest payments if the borrower's credit quality deteriorates. The lender has this right each time interest payments are due. Thus, we interpret this as a portfolio of options where the maturity of each option corresponds to an interest payment date. Denote the time 0 market value of the fixed rate loan and the put option portfolio by F_0^I and P_0, respectively. Using expression (7), $P_0 = \sum_{i=1}^{m+n} \mathcal{B}(\xi, B_i)(c_{i+1} - c_i)$. Thus, the time 0 market value of the interest increasing PSD loan L_0^I is given by $L_0^I = F_0^I + P_0$.

An interest decreasing PSD contract is, from a lender's point of view, equivalent to a fixed rate loan plus a portfolio of short call options. The call options give the borrower rights to pay reduced interest rates, at any interest payment date, if its credit quality improves. Since this right is held by the borrower, the lender is short in these options. Denote the time 0 market value of the fixed rate loan and the call option portfolio by F_0^D and C_0, respectively. Using expression (6), $C_0 = \sum_{i=1}^{m+n} \mathcal{A}(\xi, B_i)(c_{i+1} - c_i)$. The market value of the interest decreasing PSD loan L_0^D is $L_0^D = F_0^D - C_0$. The interest increasing/decreasing provisions are contractually determined and one could, thus, argue that the put and call options do not include the customary optionality at maturity included in regular options. However, any rational optionholder would exercise such options when they are in the money at maturity, so we can safely apply the option interpretation. Also, since the size of the increase or decrease in interest rate payments is independent of the underlying credit performance measure within a certain range of the credit performance measure, the put and call options are of digital type[12]. Normalizing the par value of the PSD loan to 100, the theoretically correct time 0 price is $F_0^I + P_0 = 100$ and $F_0^D - C_0 = 100$ for the interest increasing and interest decreasing cases, respectively.

A PSD contract which has both interest increasing and interest decreasing provisions can be decomposed into a portfolio of a fixed rate loan F_0^B, with fixed interest rate equal to the initial interest rate of the PSD contract, a portfolio of short digital call options, and a portfolio of long digital put options. This contract's time 0 market value can be written as $F_0^B - C_0 + P_0 = 100$.

To calculate the valuation effect that stems from the performance sensitive provisions of the example contract from the previous section, we compare the theoretical value of the PSD loan to the theoretical value of a fixed interest rate loan which pays an interest rate equal to the initial interest rate c_7 in Table IX. This value is calculated using a version of

[12] A digital option is an option whose payout is fixed after the underlying asset exceeds a predetermined threshold or strike price. In the literature these options are also commonly referred to as 'cash-or-nothing' options, see, e.g., McDonald (2006).

the pricing formula in Black & Cox (1976) modified to include finite maturity,

$$F(\xi) = c_i Z^c(\xi) + De^{-rT} Q^c(\xi) + D(1-\kappa) H^c(\xi), \tag{14}$$

where c_i is the initial payment interest rate. $Q^c(\xi)$, $H^c(\xi)$ and $Z^c(\xi)$ are given by expressions (17), (20) and (21), respectively. In this example $c_i = c_7$.

Using expression (14) and contractual terms from Table IX, the time 0 market value of a fixed interest loan is 95.86, and, thus, there is a reduction in value from adding interest decreasing provisions to the loan contract, the size of which equals $95.86 - 95.44 = 0.42$. This number equals the value of the call option portfolio $C_0 = 0.42$.

The time 0 price $\hat{F}_0$ of a comparable risk-free contract with the same initial interest rate is 123.24, calculated using

$$\hat{F}_0 = \frac{c_i}{r}\left(1 - e^{-rT}\right) + De^{-rT}. \tag{15}$$

The difference of 27.80 between the market value of the risk-free contract and the PSD contract may be decomposed into 27.38 due to default risk and 0.42 due to the interest decreasing credit performance sensitive provision.

5 Sample Analysis

We empirically test the pricing performance of our model in the following way: Firstly, we collect contractual and borrower data. Secondly, we estimate volatility and drift from borrower data. Thirdly, we impose ad hoc assumptions regarding shocks to credit quality, implemented in our model as jumps in the amount of debt. Finally, we calibrate the model prices to par using alternative jump frequencies. Subsequently, we analyze borrower profiles and pricing errors using t-tests and probit regressions and indicate some interpretations of our results.

A Specification of Shocks to Credit Quality

We assume that shocks to credit quality are given by jumps in the amount of debt. Furthermore, we assume that Y_i, the change in borrower's total debt caused by jump i, is lognormally distributed[13] under the equivalent martingale measure Q. We assume that the expectation and variance of Y_i are $E^Q[Y] = 2$ and $\mathrm{Var}^Q[Y] = \frac{1}{12}$, respectively, where superscript Q indicates the measure under which these quantities are calculated. The assumed value $E^Q[Y] = 2$ implies that the total amount of debt doubles, in risk-adjusted expectation, in case of a jump. We define 10 scenarios with different jump risk, assuming that the risk-adjusted jump intensity λ varies from 0.05 to 0.5 in steps of 0.05. These intensities correspond to the following risk-adjusted frequencies, interpretable as, on average, one jump every 20, 10, $6\frac{2}{3}$, 5, 4, $3\frac{1}{3}$, $2\frac{6}{7}$, 2.5, $2\frac{2}{9}$, 2 'risk-adjusted years', respectively. The jump inten-

[13] The specific distributional assumption of the jump size is not important. We get similar quantitative results using alternative distributions with the same numerical values of the first two moments.

sities could, in principle, be estimated from data if we assumed that the process for debt was stationary and we had sufficiently long time series, as well as the market risk premia for jump risk (size and frequency). We use jumps in the total amount of debt as a proxy for shocks to credit quality. Note that we do not analyze a borrower's optimal choice of capital structure.

B Test Statistics

The market price of a PSD loan at time 0 is given from our model as an expectation, see expression (8). Denote the price of PSD contract i, based on estimated input parameters and assumptions regarding jumps as described in the previous subsection, by $\hat{L}_i$. Our first test statistic is the sample average of the time 0 market values, defined as

$$\bar{L} = \sum_{i=1}^{N} \hat{L}_i,$$

where N is the number of observations. If our model is correct, the time 0 market value of each sample loan should be equal to the loan's normalized face value (100). This fact implies that also the average of the model's time 0 market values should be equal the normalized face value. Our second test statistic, M, is a measure of sample dispersion and is defined as the square root of the squared pricing errors (relative to the normalized face value). Here, M is given by

$$M = \sqrt{\frac{\sum_i^N \left(\hat{L}_i - 100\right)^2}{N}}. \tag{16}$$

If our pricing model correctly prices each individual loan contract, the numerical value of M would be zero. This measure is related to the standard deviation measure which uses the sample mean, instead of some theoretical value, as benchmark. Similar to standard deviation, our measure gives higher weight to observations further away from the benchmark.

C Sample Results

Table X reports the average time 0 model price ($\bar{L}$), as well as M (dispersion), separately for the full sample, interest increasing contracts, interest decreasing contracts, and contracts containing both features. Row 1 reports the results assuming no jumps in debt value from our closed form solution in expression (11). The remaining values have been estimated using standard Monte Carlo simulation techniques.

For each of the 3,052 PSD contracts in our sample we collect contractual parameters and the risk free rate (r), and estimate the drift (μ), and volatility (σ) as explained in Section 4.A. For the liquidation cost parameter we use $\kappa = 0.27$ for all contracts (see discussion in Section 4.A and Altman et al. (2004)). Subsequently, we calculate 10 000 simulations of expression (8) in a C++ program. We discretize time into 100 subperiods per year. To reduce variance we use the standard antithetic variate technique and (see, e.g., Glasserman (2004)), in addition, we apply the formula in expression (11) as control variate in the cases with jumps. The average pricing difference between the closed form values and the simulated values in the case without jumps is 0.025%[14].

The results from the case of no jumps overprice PSD contracts, with a sample average price of 107.2. Considering each subsample, interest increasing contracts are overpriced on average by 9.8% whereas contracts with both features are overpriced on average by 6.1%. For interest decreasing contracts the model produces a small underpricing by on average 0.7%, and average estimated contract values, $\bar{L}$, which are not significantly different from 100. The dispersion, M, of prices ranges from 12.2 to 17.6 for the various subsamples.

Rows 2 - 10 in Table X report the average price and dispersion measure for alternative values of the jump intensity λ from 0.05 to 0.5. As expected, average prices are decreasing in jump intensity. For the full sample $\lambda = 0.2$ produces average loan price closest to par value, with an underpricing by only 0.7%. Price dispersion is smallest for $\lambda = 0.15$. For interest increasing contracts $\lambda = 0.4$ produces the sample average price closest to par value. For this category the smallest M is produced for $\lambda = 0.35$. Interest decreasing contracts are priced closest to par value for $\lambda = 0$, although the model dispersion is smallest for $\lambda = 0.1$. Contracts with both interest increasing and interest decreasing features have the lowest M for $\lambda = 0.15$, but the average price is closest to par value for $\lambda = 0.1$. For all three subsamples the value of λ which produces the value closest to par value is close to the value of the λ which produces the lowest M. Figure 3 shows the distributions of loan prices for the full

[14]The total computer time for 3,052 contracts is about 6 hours and 20 minutes (2.8 GHz Intel Core i7 processor).

Sample Results

Table X: Table shows the time 0 average loan price ($\bar{L}$), as well as the price dispersion (M) for various jump intensities of debt. The results are reported for the full sample, interest increasing PSD contracts, interest decreasing PSD contracts and contracts with both features, respectively. The superscript $^{\$}$ indicate that the sample average is not significantly different from 100, at a 5% significance level.

	Full Sample		**Interest Increasing**		**Interest Decreasing**		**Both**	
λ	$\bar{L}$	M	$\bar{L}$	M	$\bar{L}$	M	$\bar{L}$	M
0.00	107.199	13.272	109.782	12.184	99.269$^{\$}$	17.605	106.089	13.149
0.05	104.993	11.634	108.992	11.498	95.311	15.264	102.580	10.544
0.10	102.934	10.738	107.999	10.781	92.137	14.519	99.482$^{\$}$	9.305
0.15	101.022	10.413	106.879	10.104	89.543	14.744	96.755	9.226
0.20	99.249	10.494	105.664	9.524	87.438	15.453	94.366	9.893
0.25	97.618	10.842	104.418	9.097	85.708	16.323	92.273	10.912
0.30	96.105	11.346	103.150	8.840	84.266	17.215	90.429	12.045
0.35	94.712	11.935	101.883	8.770	83.074	18.041	88.817	13.168
0.40	93.430	12.559	100.646	8.859	82.055	18.809	87.404	14.228
0.45	92.245	13.189	99.438	9.098	81.215	19.470	86.151	15.209
0.50	91.159	13.806	98.274	9.445	80.492	20.064	85.060	16.094

sample and the three subsamples, using the λ values from the calibration.

From Table X we see that introducing jump risk in the debt process has large effects, both in terms of the level of prices, but also in terms of the dispersion of prices. This observation suggests that the risk of a shock to credit quality might be important when valuing PSD contracts. Observe that including a small jump risk decreases dispersion compared to no jump risk across all subsamples.

D Comparing Borrower Characteristics of Interest Increasing and Decreasing Contracts

We know from Subsection 3.B.2 that firms with interest increasing PSD contracts seem to be of an overall better credit quality than firms with interest decreasing PSD contracts. In particular, the average (median) leverage of the former group is 0.19 (0.14), versus the latter group at 0.42 (0.40), cf. Tables XVI and XVII. Table XI displays the correlation between

Correlations Between Loan Type and Indicators of Credit Quality

Table XI: Table shows the correlation between a dummy variable called 'type' and several indicators of borrower credit quality. The dummy variable takes the value 1 if the PSD loan is of interest increasing type, and the value 0 if the loan is of interest decreasing type. All correlation coefficients are significant at a 1% significance level. The default probabilities are simulated.

Type	Asset Volatility	Leverage	ROCE	DTD	Payout	Default Probability
Correlation	-0.2864	-0.4462	0.1466	0.2569	-0.1538	-0.0898

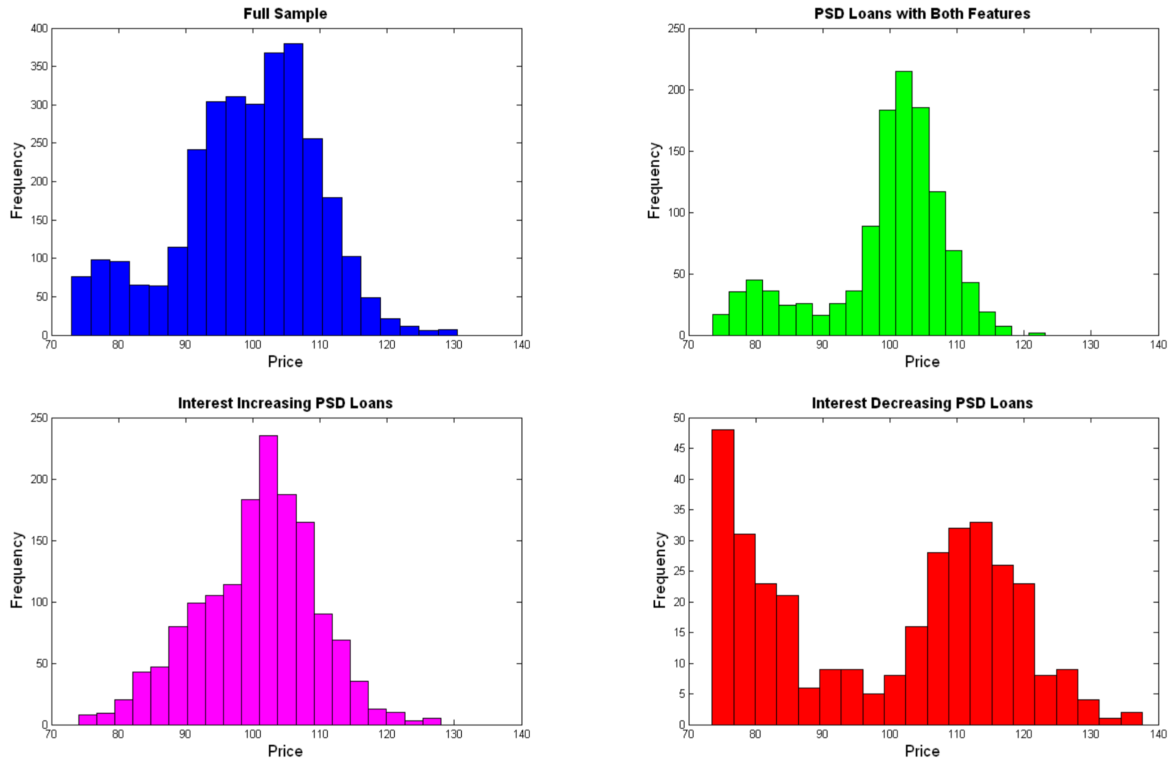
several indicators of credit quality and a dummy variable taking the value 1 if the loan is of interest increasing type, and the value 0 if the loan is of interest decreasing type. Table XI further supports the idea that borrowers with a higher credit quality select interest increasing PSD contracts. Examining credit ratings for the two subsamples, we also find that 15% of borrowers using interest increasing PSD contracts are rated investment grade, compared to only 6% of the borrowers using interest decreasing PSD contracts. It is, therefore, likely that the former category of borrowers have larger debt capacity and, hence, have a higher probability of increasing debt in the future. The latter category of firms are likely to have lower debt capacity implying that they have less chance of obtaining additional debt financing in the future. Note that borrowers with higher credit quality normally could expect to obtain more competitive terms on their debt. The calibration of our model shows a significant difference between interest increasing and interest decreasing contracts regarding shocks to credit quality. From Table X we see that, in a model disregarding shocks to credit quality, interest increasing PSD contracts are valued 9.8% above par. Our results establish the puzzle that the most financially sound borrowers choose interest increasing PSD contracts, which are priced reflecting a substantially larger risk of reduced credit quality, compared to interest decreasing PSD contracts. These findings indicate that banks price interest increasing PSD contracts by rationally taking into account the probability that borrowers will increase the amount of debt, and is not inconsistent with the hypothesis of Manso et al. (2010), that high quality firms use interest increasing PSD contracts to signal quality.

E Analyzing Pricing Errors

We analyze the pricing errors based on our earlier calibration. We include other potential factors, not necessarily included in our pricing model, that may affect prices of PSD loans. Following Eom et al. (2004), we compare loans that are overpriced to loans that are under-priced. We use the variables asset volatility, payout ratio, leverage, and distance-to-default to capture the risk characteristics of the borrowers. We, furthermore include log of sales as a proxy for borrower size and ROCE to measure borrower profitability. We also include loan specific characteristics like amount, maturity, and initial credit spread. In addition, we include PSD contract characteristics like number of non-absorbing barriers and credit perfor-

Distribution of Calibrated Contract Prices

Figure 3: The histograms show the distribution of time 0 theoretical market values of the full sample of PSD contracts ($\lambda = 0.2$), interest increasing PSD contracts ($\lambda = 0.4$), interest decreasing PSD contracts ($\lambda = 0$), and contracts with both features ($\lambda = 0.1$), respectively.



mance sensitivity[15]. To measure the relative importance of PSD in the firm's debt structure we also use the ratio of the new PSD loan to other debt the company had at the time of issue.

For each of the chosen variables we initially use a t-test to test whether or not there are any difference in means of the variables when comparing overpriced loans to underpriced loans. Table XII reports the differences in means from this test for the full sample and the subsamples of interest increasing, interest decreasing and loans with both features, respec-

[15] To measure the credit performance sensitivity of a loan contract we define a variable called 'Diff', which is the maximum credit spread less the minimum credit spread as specified in the PSD contract.

tively. As an example, the large negative difference in asset volatility indicates that overpriced loans are granted to firms with lower average asset volatility compared to underpriced loans. Note also that these firms have significantly lower leverage, higher distance-to-default and higher return. Overall, our model tends to overprice loans given to firms which are less risky and more profitable, as discussed above. The two categories of loans also differ in amount, initial credit spread and maturity, with overpriced loans having smaller amounts, shorter maturities and larger initial credit spreads. The performance sensitivity parameter does not seem to influence pricing, but overpriced loans have, on average, fewer barriers specified in the contract. For overpriced loans the relative importance of the PSD in the debt structure is much larger, probably reflecting the fact that these firms tend to have lower total debt, and higher probability of increased borrowing. The borrower size variable is insignificant for the full sample and the subsamples of interest increasing and interest decreasing contracts. By looking at the three subsamples, we observe that the signs and the significance of the test statistics comparing over- versus underpriced contracts are similar to the full sample. However, interest decreasing contracts differ in that only asset volatility is of statistical and economic difference between over- and underpriced loans. This observation is somewhat puzzling given the distribution of loan values shown in Figure 3.

The results in Table XII point to a number of systematic differences which may affect the pricing of PSD loans. A combination of factors may lead to higher or lower pricing errors, and therefore that an analysis in a multivariate regression setting is more appropriate. We

Comparing Means for Over- and Under-priced Loans

Table XII: Table shows the difference in means of borrower, loan, and PSD characteristics when comparing overpriced PSD loans to underpriced PSD loans. A negative value suggests that overpriced loans have a smaller mean compared to underpriced loans. A superscript of *, ** and *** indicate a significance level of 5%, 1% and 0.1%, respectively.

	T-Statistics			
Variables	**Full Sample** ($\lambda = 0.2$)	**Interest Increasing** ($\lambda = 0.4$)	**Interest Decreasing** ($\lambda = 0.0$)	**Both** ($\lambda = 0.1$)
Borrower Characteristics				
Asset Volatility	-0.13***	-0.08***	-0.34***	-0.21***
Leverage (Debt/(Debt + Equity))	-0.20***	-0.17***	-0.02	-0.08***
ROCE (%, quarterly)	0.02***	0.01***	0.004	0.004***
DTD	90.31***	140.53***	2.00***	7.00***
Payout Ratio	-0.003***	-0.002***	-0.000	-0.005***
Size (MUSD)	25.97	-74.89	207.13	-192.78*
Loan Characteristics				
Loan Amount (MUSD)	-47.54***	-45.47***	40.30	-83.53***
Loan Maturity	-0.39***	-0.89***	0.22	-0.20*
Initial Credit Spread	-4.66	8.43	2.45	15.66
PSD Characteristics				
# Barriers	-0.27***	-0.15**	-0.56***	-0.20**
Diff	-0.03	0.02	-0.10*	0.02
PSD/Debt	0.23***	0.23***	0.02	0.04***

define a new dummy variable 'overpricing', which is equal to 1 if the loan is overpriced, i.e., if $\hat{L}_i > 100$ and 0 if the loan is underpriced, i.e., $\hat{L}_i < 100$, using subsample calibrated λ values. We then use a probit model to regress this dummy variable on our set of explanatory variables, as well as year dummies for the year the loan is granted. Table XIII reports the coefficients from the probit regression and the corresponding standard errors. A regression of the whole sample shows that if a loan is of interest increasing type the probability that it will be overpriced is higher. We also see that the probability of overpricing increases with the initial credit spread paid on the loan and the performance sensitivity of the PSD contract, whereas it decreases with maturity. Our model also overprice loans given to safer borrowers, as seen by the negative coefficients on payout-ratio and asset volatility, and on the positive coefficients on distance-to-default. No other explanatory variables are significant.

Motivated by the fundamental differences in borrower characteristics and contract features, we also run regressions for each of the three subsamples. The other regressions in Table XIII therefore report the results using the model specification that produce the sample average closest to par value for the subsample. From Column 2 we see that the probability that interest increasing PSD contracts will be overpriced is inversely related to leverage, loan amount and maturity, and positively related to initial credit spread paid on the loan, the relative size of the loan and the distance-to-default. The negative coefficient on current leverage supports our earlier interpretation that overpricing may be explained by jump risk in the amount of debt. A borrower with high leverage today is less likely to increase leverage in the future. However, we see that by introducing jump risk in the modeling of interest increasing contracts the partial significance of leverage in explaining overpricing in Table X nearly disappears. All other explanatory variables are insignificant. The probability that an interest decreasing PSD contract is overpriced is inversely related to the asset volatility, whereas, it is positively related to initial credit spread and distance-to-default. The results are similar when analyzing the subsample of PSD contracts containing both interest increasing and interest decreasing features, except that the performance sensitivity of the contract and the number of barriers also become significant. Overall the only two variables that get significant coefficients across all 4 regressions are the initial credit spread paid on the loan and the distance-to-default. This observation indicates that the pricing of the PSD loans is crucially related to the initial credit spread, and that the performance sensitive features added to the loan contract are of lesser importance for any over- or underpricing. We also see that loans with a high contractual distance-to-default tend to be more overpriced, i.e., that the model tend to underestimate the probability of default for the safest firms. This is both consistent with high jump risk and earlier results regarding the performance of structural credit models as discussed above.

F Interpretation of the Empirical Results

The results and the subsequent analysis in the previous subsections point to several interesting insights.

Firstly, we price PSD contracts with, in expectation, downward jumps in the credit performance measure, interpreted more generally as shocks to credit quality. We have chosen to implement the jumps as changes in the borrower's total debt, allowing for the use of a continuous cash flow process. Our performance measure is a ratio between these two quantities, and we naturally model each quantity separately. Practitioners tell us that the Debt/CF performance measure is also used to discipline borrowers from additional borrowing. The risk of shocks to credit quality could be modeled in several ways, but our choice of adding jump risk to the amount of debt is analytically tractable.

Secondly, the identical jump intensity does not calibrate both interest increasing and

Joint Test of Variables Affecting Pricing Errors

Table XIII: Estimation results from a Probit regression of a dummy variabel taking the value 1 if the PSD loan is overpriced and 0 otherwise on borrower characteristics like asset volatility, leverage, ROCE, distance-to-default (DTD), payout ratio and size (proxied by log of sales). Furthermore we also use explanatory variables specific to the loan contracts like log of loan amount, maturity, initial credit spread paid on the loan, number of non-absorbing barriers, a variable (diff) measuring the performance sensitivity in the contract, as well as the relative size of the PSD loan to existing debt. We also include calendar year dummy variables and two dummy variables capturing whether the loan is of interest increasing or interest decreasing type. The regression is run using the best fitted pricing model for the full sample as well as the three subsamples, as indicated by the λ values. We report coefficients and robust standard errors in parenthesis.

	Full Sample ($\lambda = 0.2$)	**Interest Increasing** ($\lambda = 0.4$)	**Interest Decreasing** ($\lambda = 0.0$)	**Both** ($\lambda = 0.1$)
Borrower Characteristics				
Asset Volatility	-1.473***	-0.286	-14.511***	-6.328***
	(0.275)	(0.423)	(3.688)	(0.676)
Leverage	-0.072	-0.872*	-0.653	0.187
	(0.258)	(0.443)	(0.607)	(0.361)
ROCE	1.738	2.230	2.886	2.017
	(1.346)	(1.938)	(5.144)	(2.193)
Distance-to-default	0.226***	0.063***	0.539***	0.237***
	(0.014)	(0.008)	(0.164)	(0.030)
Payout Ratio	-0.255***	-0.102	-0.022	-0.111
	(0.060)	(0.056)	(0.141)	(0.061)
Size	-0.063	0.099	-0.106	-0.082
	(0.047)	(0.058)	(0.137)	(0.061)
Loan Characteristics				
Amount	-0.006	-0.139*	-0.217	-0.087
	(0.053)	(0.067)	(0.155)	(0.069)
Maturity	-0.054*	-0.309***	0.083	-0.011
	(0.027)	(0.037)	(0.090)	(0.038)
Initial Credit Spread	0.007***	0.009***	0.011***	0.007***
	(0.0006)	(0.0009)	(0.002)	(0.0009)
PSD Characteristics				
Number of Barriers	-0.046	-0.097	-0.079	-0.119*
	(0.049)	(0.066)	(0.149)	(0.060)
Diff	0.547***	0.263	0.485	0.816***
	(0.147)	(0.184)	(0.595)	(0.199)
PSD/Debt	0.301	1.116***	0.108	0.020
	(0.230)	(0.279)	(0.706)	(0.323)
Interest Increasing	1.010***			
	(0.094)			
Interest Decreasing	-0.222			
	(0.160)			
N	3,052	1,520	342	1,190
Pseudo R^2	0.720	0.638	0.6489	0.538

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

interest decreasing PSD contracts to par values. This fact suggests that there is a fundamental difference between the typical borrower using interest increasing PSD and interest decreasing PSD, respectively. According to our model, the market prices interest decreasing contracts as if no jump risk is present. Interest increasing contracts are priced consistent with a risk-adjusted jump intensity of 0.4, suggesting that jump risk is important in the pricing of these contracts. Based on these results we believe that the pricing of interest increasing PSD contracts reflects that the companies using these contracts probably have a larger debt capacity, and therefore have a higher probability of adding new debt to their capital structure in the future. The pricing of interest decreasing contracts reflects that companies using such contracts probably have a smaller debt capacity and a lower probability of additional debt funding in the future. This interpretation is supported by comparing borrower characteristics for the subsamples. Our findings support the signaling hypothesis of Manso et al. (2010), and add to the understanding of their findings. High quality borrowers might be able to signal quality to the market by using interest increasing PSD loans. High quality borrowers acknowledge the presence of risk of reduced credit quality and find the conditions of interest increasing contracts acceptable. Low quality borrowers which have less debt capacity find interest increasing contracts too expensive. Our results are in contrast to the findings of Asquith et al. (2005), who find that interest decreasing PSD loans tend to be used when costs related to adverse selection problems are larger.

Thirdly, our analysis of pricing errors shows that borrower's contractual distance-to-default is particularly important. A large DTD value increases the probability that our model calibrated with jump risk overprice any type of PSD contract. One way to compensate this problem is to increase the probability of jumps in the underlying performance measure. In our class of PSD contracts, a jump in the credit performance measure is modeled by a jump in borrower's total debt. Interest increasing loans, used by borrowers with a large DTD, require a higher jump intensity in debt to be priced correctly, on average. Our findings are consistent with other empirical tests of structural debt pricing models, see, e.g., Eom et al. (2004) and Huang & Huang (2003).

6 Conclusions and Further Research

A Conclusions

Performance sensitive debt is a large class of debt contracts where the interest payment is contractually defined to change according to some predetermined credit performance measure. Our model prices PSD contracts with cash flow and debt based credit performance measures. Based on financial valuation theory, we derive a valuation model and test its performance empirically on a sample of 3,052 PSD contracts using contractual, borrower and market data, as well as calibrated jump frequencies. Our model incorporate finite maturity, jump risk in the borrower's total amount of debt, as well as exogenous contract-specific default covenants. In the special case of no jump risk, we also derive a closed form solution for the market price of the contract.

In our empirical analysis, we show that interest decreasing contracts are priced on average closest to par value using our closed form model disregarding shocks to credit quality, whilst for interest increasing contracts a substantial amount of credit quality risk calibrates the model price to par value. This result fits well with characteristics of borrowers using such contracts which have a larger debt capacity due to an overall higher credit quality, e.g., represented by a larger distance-to-default, and a lower current leverage. A comparison of borrower characteristics for the two subsamples confirms this interpretation. Our empirical results show that PSD contracts are priced recognizing the risk of future increases in debt with a potentially negative effect on current loan value. These results also contribute to a better understanding of the signaling hypothesis presented in Manso et al. (2010). High quality borrowers, which are less credit constrained, may signal quality through the use of interest increasing PSD loans. Low quality borrowers, which are more credit constrained, choose not to mimic this behavior since they have to incur a cost related to the lenders assessment of future jump risk, i.e., risk of increased leverage. Instead they would prefer interest decreasing PSD loans.

In addition to an increased understanding of PSD loans, we also contribute to the more general literature which empirically test structural debt models. Our results support earlier findings that structural debt models require jump risk in order to calibrate market prices on loans given to the safest borrowers.

B Future Research

An important implementation issue for PSD contracts is the observability and verifiability of the underlying performance measure. For accounting based performance measures the observability is determined by the borrower's external financial reporting frequency. These reports present a delayed and discretized measure of the borrower's credit quality. Any valuation effects of these implementation issues are not included in our analysis, but might be an avenue for future research.

Our analysis suggests that borrowers using interest decreasing PSD contracts have higher leverage and, correspondingly, lower debt capacity. This result indicate that interest decreasing PSD loans might be used when problems related to debt overhang are severe. Intuitively, a debt contract that promises reduced interest rate payments when firm performance increase, might incentivize borrowers to invest earlier.

Although debt contracts have default clauses, a violation of those clauses does not necessarily trigger bankruptcy in reality. However, the lender may instead instead renegotiate the contract. Our pricing approach is based on the assumption that hitting the bankruptcy barrier automatically trigger default. There is a recent literature built around the idea that debt can be renegotiated, see ,e.g., Hackbarth, Hennessy, & Leland (2007) or Garleanu & Zwiebel (2009). Future research may take this idea into account.

Our finding that interest increasing PSD loans are priced reflecting a high risk of shocks to credit quality is also a potential avenue for further studies. This research idea includes a closer review of loan covenants as well as general ex post realizations.

Our model is based on a constant risk free interest rate. Clearly, our analysis can be extended to a model with stochastic interest rates.

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Appendix A Closed Form Solutions in the Case of No Jumps

Let $N(\cdot)$ denote the cumulative standard normal distribution function.

A Survival probability

The survival probability $Q^c(\xi)$ is given by

$$Q^c(\xi) = Q(\tau > T) = N(d_1) - \left(\frac{\xi}{C}\right)^{\alpha-\beta} N(-d_2), \tag{17}$$

where

$$d_1 = \frac{\ln\left(\frac{\xi}{C}\right) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{\xi}{C}\right) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$\alpha = \frac{1}{\sigma^2}\left(\frac{1}{2}\sigma^2 - \mu + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right), \tag{18}$$

and

$$\beta = \frac{1}{\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right). \tag{19}$$

B Time 0 market price of 1 upon default

The time 0 market value $H^c(\xi)$ of a security paying 1 when the default barrier is hit is given by

$$H^c(\xi) = E^Q[e^{-r\tau}1\{\tau \leq T\}] = e^{b(u-w)}N\left(\frac{b - wT}{\sqrt{T}}\right) + e^{b(u+w)}N\left(\frac{b + wT}{\sqrt{T}}\right), \tag{20}$$

where $u = (\mu - (1/2)\sigma^2)/\sigma$, $w = \sqrt{u^2 + 2r}$, and $b = \ln(C/\xi)/\sigma$, see Appendix B.2 in Lando (2004).

C Time 0 market price of finite unity interest payments

$$Z^c(\xi) = E^Q\left[\int_0^{\tau\wedge T} e^{-rs}ds\right] = \frac{1}{r}\left[1 - e^{rT}Q^c(\xi) - Q_l^\beta(\frac{\xi}{C})^{-\beta}\right], \tag{21}$$

where β and $Q^c(\xi)$ are given in expressions (19), and (17), respectively. Also,

$$Q_g^\beta = Q^\beta(\tau > T) = N\left(d_1^\beta\right) - \left(\frac{\xi}{C}\right)^{\alpha+\beta} N\left(-d_2^\beta\right), \tag{22}$$

where

$$d_1^\beta = \frac{\ln\left(\frac{\xi}{C}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$
$$d_2^\beta = \frac{\ln\left(\frac{\xi}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

and α is given in expression (18). Furthermore,

$$Q_l^\beta = Q(\tau < T) = 1 - Q_g^\beta, \tag{23}$$

where Q_g^β is given in expression (22).

D Time 0 market values of above unity annuities

The following result is from expression (17) in Mjøs & Persson (2010). Define

$$x = \frac{\alpha}{\alpha+\beta},$$

$$y = \frac{\beta}{\alpha + \beta}.$$

Now,

$$\mathcal{A}_a^c(\xi, B) = \gamma_a(\xi, B)/r, \tag{24}$$

where

$$\gamma_a(\xi, B) = 1 - x\left(\frac{\xi}{B}\right)^{-\beta}(1 - Q_{gg}^{\beta}(B)) - y\left(\left(\frac{\xi}{B}\right)^{\alpha} Q_{lg}^{\alpha}(B) + \left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta} Q_l^{\beta}\right) - e^{-rT} Q_{gg}(B).$$

The probabilitites

$$Q_{gg}^{\beta}(B) = Q^{\beta}(\xi_T > B, \tau > T) = N\left(d_3^{\beta}\right) - \left(\frac{\xi}{C}\right)^{\alpha+\beta} N\left(-d_4^{\beta}\right), \tag{25}$$

where

$$\begin{aligned} d_3^{\beta} &= \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\ d_4^{\beta} &= \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \end{aligned}$$

and

$$Q_{lg}^{\alpha}(B) = Q^{\alpha}(\xi_T < B, \tau > T) = N\left(d_1^{\alpha}\right) - N\left(d_3^{\alpha}\right) + \left(\frac{\xi}{C}\right)^{-(\alpha+\beta)}\left(N\left(-d_4^{\alpha}\right) - N\left(-d_2^{\alpha}\right)\right), \tag{26}$$

where

$$
\begin{aligned}
d_1^\alpha &= \frac{\ln\left(\frac{\xi}{C}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_2^\alpha &= \frac{\ln\left(\frac{\xi}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_3^\alpha &= \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_4^\alpha &= \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.
\end{aligned}
$$

Furthermore,

$$
Q_{gg}(B) = Q(\xi_T > B, \tau > T) = N\left(d_3\right) - \left(\frac{\xi}{C}\right)^{\alpha-\beta} N\left(-d_4\right), \tag{27}
$$

where

$$
\begin{aligned}
d_3 &= \frac{\ln\left(\frac{\xi}{B}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
d_4 &= \frac{\ln\left(\frac{\xi}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.
\end{aligned}
$$

From the expressions (12) and (15) in Mjøs & Persson (2010) it follows that

$$
\mathcal{A}_b^c(\xi, B) = \gamma_b(\xi, B)/r, \tag{28}
$$

where

$$\gamma_b(\xi,B) = x\left(\frac{\xi}{B}\right)^{-\beta} Q^{\beta}_{gg}(B)+y\left(\left(\frac{\xi}{B}\right)^{\alpha}(1-Q^{\alpha}_{lg}(B))-\left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta}Q^{\beta}_{l}\right)-e^{-rT}Q_{gg}(B),$$

where $Q^{\beta}_{gg}(B)$, $Q^{\alpha}_{lg}(B), Q^{\beta}_{g}, Q_{gg}(B)$ are given in expressions (25), (26), (22), (27), respectively.

E Time 0 market values of below unity annuities

The following result is from expression (18) in Mjøs & Persson (2010).

$$\mathcal{B}^{c}_{a}(\xi,B) = \eta_a(\xi,B)/r, \tag{29}$$

where

$$\eta_a(\xi,B) =$$

$$x\left(\frac{\xi}{B}\right)^{-\beta}(1-Q^{\beta}_{gg}(B))+y\left(\left(\frac{\xi}{B}\right)^{\alpha}Q^{\alpha}_{lg}(B)+\left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta}Q^{\beta}_{l}\right)-e^{-rT}Q_{lg}(B)-\left(\frac{\xi}{C}\right)^{-\beta}Q^{\beta}_{l},$$

where $Q^{\beta}_{gg}(B)$, $Q^{\alpha}_{lg}(B)$, Q^{β}_{l} are given in expressions (25), (26), (23), respectively. Here,

$$Q_{lg}(B) = Q^{c}(\xi) - Q_{gg}(B), \tag{30}$$

where $Q^{c}(\xi)$ and $Q_{gg}(B)$ are given in the expressions (17) and (27), respectively. From the expressions (13) and (16) in Mjøs & Persson (2010) it follows that

$$\mathcal{B}^{c}_{b}(\xi,B) = \eta_b(\xi,B)/r, \tag{31}$$

where

$$\eta_b(\xi, B) =$$

$$1 - x\left(\frac{\xi}{B}\right)^{-\beta} Q_{gg}^{\beta}(B) - y\left(\left(\frac{\xi}{B}\right)^{\alpha}(1 - Q_{lg}^{\alpha}(B)) - \left(\frac{C}{B}\right)^{\alpha}\left(\frac{\xi}{C}\right)^{-\beta} Q_l^{\beta}\right) - e^{-rT} Q_{lg}(B) - \left(\frac{\xi}{C}\right)^{-\beta} Q_l^{\beta},$$

where $Q_{gg}^{\beta}(B)$, $Q_{lg}^{\alpha}(B)$, $Q_{lg}(B)$, Q_l^{β} are given in expressions (25), (26), (30), (23), respectively. Observe that $r(\gamma_a(\xi, B) + \eta_a(\xi, B)) = r(\gamma_b(\xi, B) + \eta_b(\xi, B)) = Z^c(\xi)$, the value of an above- and a below annuity should add to the value of a regular annuity, no matter if the initial value ξ is above or below a given B.

Appendix B Tables and Graphs

Industry Distribution of PSD

Table XIV: This table shows the industry distribution of borrowers of performance sensitive debt across broad industry classes. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010 (N=25,602).

Industry	**% of total**
Banks	0.6 %
Corporates	76.2 %
Government	0.3 %
Media/Communications	11.3 %
Non-bank Financial Inst.	5.1 %
Utilities	5.1 %
Others	1.3 %

Descriptive Loan Statistics of Database

Table XV: This table shows discriptive statistics for maturity and loan amounts for loans containing PSD features and for loans not containing PSD features. Datasource: Thomson Reuter's Dealscan database for the years 1993-2010.

	Maturity (Years)		**Facility Amount (MUSD)**	
	PSD Facilities	**Non-PSD Facilities**	**PSD Facilities**	**Non-PSD Facilities**
Mean	8.7	9.4	373	195
Median	10	8	150	60
St.Dev	4.6	8.3	887	602
Min	0.17	0.17	0	0
Max	64.2	146.8	30,000	61,607
N	25,602	192,602	25,602	192,602

Summary Statistics for Interest Increasing PSD Contracts

Table XVI: This table shows summary statistics for various model input parameters and firm characteristics for the **interest increasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics						
Company Sales (MUSD)	1,034	381	4,465	3.28	159,098	1,520
ROCE (%,quarterly)	5.01	4.27	6.24	-12.86	112.17	1,520
Leverage (Debt/(Debt + Equity))	0.19	0.14	0.18	0.00	0.99	1,520
PSD Loan/Total Debt	0.59	0.58	0.27	0.00	1.00	1,520
Drift of cash flow	0.024	0.027	0.014	-0.09	0.058	1,520
Volatility of cash flow	0.084	0.038	0.13	0.02	0.80	1,520
Loan Characteristics						
Loan Amount (MUSD)	164	100	350	0.06	10,700	1,520
Maturity (Years)	4.47	5.00	1.72	0.08	21	1,520
All-In-Spread (Bp)	186	175	89	23	750	1,520
# of Barriers	3.2	3	1.20	1	7	1,520
Distance-to-default	94.1	28.3	149.7	0.03	1092.6	1,520

Summary Statistics of Interest Decreasing PSD Contracts

Table XVII: This table shows summary statistics for various model input parameters and firm characteristics for the **interest decreasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics						
Company Sales (MUSD)	880	475	1,217	9	10,877	342
ROCE (%,quarterly)	2.77	2.60	3.79	-4.79	63.68	342
Leverage (Debt/(Debt + Equity))	0.42	0.40	0.19	0.01	0.90	342
PSD Loan/Total Debt	0.30	0.27	0.18	0.02	1	342
Drift of cash flow	0.02	0.022	0.013	-0.060	0.047	342
Volatility of cash flow	0.20	0.08	0.23	0.02	0.80	342
Loan Characteristics						
Loan Amount (MUSD)	184	100	278	2	3,000	342
Maturity (Years)	4.91	5.00	1.72	0.92	10.08	342
All-In-Spread (Bp)	219	225	87	50	600	342
# of Barriers	2.95	3	1.35	1	7	342
Distance-to-default	1.21	0.34	2.15	0.00	24.88	342

Summary Statistics for PSD Contracts Containing Both Increasing and Decreasing Provisions

Table XVIII: This table shows summary statistics for various model input parameters and firm characteristics for sample **PSD contracts containing both interest increasing and interest decreasing provisions**. The loan contracts are granted in the period 1993 - 2010. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics						
Company Sales (MUSD)	954	466	1,550	1.08	19,424	1,190
ROCE (%,quarterly)	3.29	3.06	2.55	-17.09	24.57	1,190
Leverage (Debt/(Debt + Equity))	0.33	0.29	0.19	0.01	0.99	1,190
PSD Loan/Total Debt	0.40	0.39	0.21	0.00	1	1,190
Drift of cash flow	0.022	0.024	0.013	-0.07	0.053	1,190
Volatility of cash flow	0.14	0.06	0.18	0.02	0.80	1,190
Loan Characteristics						
Loan Amount (MUSD)	201	125	277	1.00	4,500	1,190
Maturity (Years)	4.53	5.00	1.60	0.25	10.17	1,190
All-In-Spread (Bp)	180	175	81	25	450	1,190
# of Barriers	3.93	4	1.13	2	8	1,190
Distance-to-default	5.81	2.59	9.29	0.01	78.73	1,190

Variable Descriptions

Table XIX: Table describes the variables used in the probit regressions in Table XIII, and displays their respective means.

Variable	**Description**	**Means**			
		Full Sample	Increasing	Decreasing	Both
Log of Borrower Sales	Logarithm of borrowers sales (in MUSD)	19.87	19.79	19.91	19.97
Borrower Sales	*Borrower sales (in MUSD)*	*986*	*1,034*	*880*	*954*
Asset Volatility	Estimated annual cash flow (asset) volatility	0.12	0.084	0.203	0.139
ROCE	Borrower's Quarterly Return on Capital Employed Based on the last quarterly report prior to the loan issuance	4.09%	5.02%	2.77%	3.30%
(Initial) Leverage	Borrower's market leverage prior to loan issue	0.27	0.19	0.42	0.33
PSD Loan/Total Debt	Ratio of PSD loan amount to the borrower's total book value of debt (including the PSD loan) at the time of loan issuance	0.48	0.59	0.30	0.40
Log of Loan Amount	Logarithm of loan amount (in MUSD)	18.37	18.25	18.41	18.53
Loan Amount	*Loan amount (in MUSD)*	*181*	*164*	*184*	*201*
Maturity	Maturity of the PSD Loan (in years)	4.54	4.47	4.91	4.53
Barriers	Number of non-absorbing contractual barriers	3.46	3.21	2.95	3.93
Distance-to-Default	The distance between borrower's initial CF/Debt ratio and the contractually specified default barrier (C), normalized by borrower's cash flow volatility	49.24	94.05	1.21	5.81
Payout ratio	Variable measuring the total cash payouts made by the borrowing firm	0.013	0.012	0.017	0.013
Initial Credit Spread	The initial credit spread paid on a loan in basis points.	187	186	219	180
Diff	Variable measuring the performance sensitivity of a loan using the difference of the maximum credit spread and the minimum credit spread in the contract	91	87	80	101
Interest Increasing	Dummy variable equal to one if the loan is of interest increasing type				
Interest Decreasing	Dummy variable equal to one if the loan is of interest decreasing type				